Introduction to Algebra

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Objectives

- Formulas
- Algebra Definitions
- Combining Like Terms
- Distributive Property
- Defining an Equation
- Solving Equations involving the Four Basic Operations
- Solving Equations with More than One Step
- Solving Equations with Parentheses and Variables on Both Sides
- Solving Equations given a Formula
- Solving Formulas
- Translating English to Algebra
- Setting up Word Problems
- Scientific Notation

Formulas

- Formulas express a commonly done calculation. Letters are used to represent the values that can change. Formulas are used to be very concise with this repeated calculation.
- Evaluating formulas – plugging in the values we know and calculating (using order of operations)
- Example: This is a formula we have already used: Calculate the temperature in Celsius equivalent to 72° Fahrenheit. $C = \frac{5(F-32)}{9}$

Try Yourself

- The formula to determine the approximate weight of steel round stock in pounds is calculated using the formula: $W = \frac{.283\pi D^2 H}{4}$ where

$\pi = 3.14$

D = diameter in inches

H = length in inches

What is the weight of a 3" diameter, 20" long piece of steel round stock?

Algebra Definitions

- Discussion: What is the point of algebra? How can we use it or how have we already been using it?

- **Variable** – an unknown number represented by a letter
  - Example: a, n, x, r, C, θ, A, w, d
- **Coefficient** – number multiplied by a variable, usually in front of the variable
  - Example: 3n means n is the variable and 3 is the coefficient of n. If n = 2, 3n is 3*2 = 6. If n = 10, 3n is 3*10 = 30, etc.
- **Term** – a variable and coefficient together
- **Constant term** – a term with no variable
  - Example: 4x + 2 , 2 is the constant term
- **Algebraic expression** – several terms separated by +'s and –'s
  - Example: 5x – 2y + 8

Algebra Definitions

- Example: For $x - 2y + \frac{5z}{16} - 10$ what are the…
  - Variables:
  - Coefficients (left to right):
  - Terms:
  - Constant Term:

Combining Like Terms

- Like Terms – Terms with the same variable or variables with the same exponents
- To combine like terms, add (or subtract) their coefficients
- Example: $4a + 5a - 2a$
- Example: $5x^2 - 3x - 2x^2 + 2x + x$
- Example: $7x - 5y + 2xy - y + xy - 7x$

Try Yourself

- 1) $4k - 5j + k - 2k + 3j$
- 2) $2n + 3n^2 - 2n - n^2 + 5n^2$

Distributive Property

- a, b, and c can be variables or numbers: $a(b + c) = ab + ac$
- Basically, we are multiplying what is in front of the parentheses by everything inside the parentheses
- This works for any number of terms in the parentheses
- Example: $2(5k + 3)$
- Example: $6(x - 4)$
- Example: $-2(8m + 2)$

Distributive Property

- Example: $-4(5k + 3 - 2j)$
- Example: $5n - (4p - 6)$
- Example: $3(2a - 5) - 4$
- Example: $3x + 2x(8 - 3x) - 5x$

Try Yourself

- 1) $-2(4r - 6) + 5(2r - 1)$
- 2) $8 - 3(x - 2) - (5 - 3x)$
- 3) $4a(3a + 4) - 3a^2$

Equations

- Equation – two equal algebraic expressions
- Recall that a variable is an unknown number represented with a letter. Think of it as a placeholder for some number.
- For a number to be a solution to an equation, when you plug the number in for the variable, the two sides of the equation need to be equal.
- Example of how you can use an equation: https://www.wisc-online.com/Objects/ViewObject.aspx?ID=TMH5206

Solving Equations (One Step)

- Example: x + 2 = 7
  - What operation did we perform here to get x alone?
- Example: x – 3 = 6
  - What operation did we perform here to get x alone?

Solving Equations

- Example: 4b = 20
  - What operation did we perform here to get x alone?
- Example: $\frac{x}{5} = 6$
  - What operation did we perform here to get x alone?

Solving Equations

- Example: 4½ + a = 7
- Example: -4 = x – 6

Solving Equations

- Example: 1.7n = 1.2
- Example: $\frac{n}{-3} = 11$

Try Yourself

- 1) $\frac{k}{7} = 4$
- 2) 14.25 = x – 1.5

Try Yourself

- 3) 4 + x = -10
- 4) 5n = -17

Keys to Solving Equations

- The goal is to get the variable alone.
- Perform the opposite operation being done to the variable.
- Whatever you do to one side, you must do to the other. (Think of a scale and we have to keep the two sides balanced)
  - http://nlvm.usu.edu/en/nav/frames_asid_201_g_4_t_2.html

Solving Equations (Two Steps)

- Get all variable terms on one side and all constant terms on the other side first.
- Example: 4n – 2 = 6
- Example: 6 – 2x = 16

Solving Equations

- Example: 8 = ¾ x – 1
- Example: $\frac{5}{3} = \frac{x}{6}$

(discuss cross multiplying)

Try Yourself

- 1) $\frac{2}{9} = \frac{x}{12}$
- 2) 3x + 5 = 17

Steps to Solving Equations with Multiple Steps

1. Remove all parentheses (distribute).
2. Combine like terms on each side.
3. Get all variable terms to one side and all constant terms on the other side.
4. Divide by the coefficient.

Equations with Multiple Steps

- Example: 4x – 3 + 2x = 13 + x - 1

Equations with Multiple Steps

- Example: $6 + 7(x - 3) = 2(x + 5)$

Try Yourself

- 1) $2a + 5 - 7a = 18$
- 2) $12.1 = 5.2y - 3.4y + 5.4$

Try Yourself

- 3) $4(x + 3) + x = 5 - 2x$

Solving Equations given a Formula

- The equation to convert Celsius temperatures to Fahrenheit is $F = \frac{9C}{5} + 32$. If it is 50°F, what is the temperature in Celsius?

Try Yourself

- The formula to determine the approximate weight of steel round stock in pounds is calculated using the formula:

$W = \frac{.283\pi D^2 H}{4}$ where

$\pi = 3.14$

D = diameter in inches

H = length in inches

If the maximum weight a part can be is 15 pounds and ½" steel round stock is being used, what is the maximum length the part can be?

Solving Formulas

- What is the goal when we are solving equations? – To get the variable alone.
- Just as we've been solving equations, when we have more than one variable, the goal is still to get the variable that we are solving for alone.

Solving Formulas

- Example: For area of a rectangle, A = l•w
  - If the area of a rectangle is 15 and the length is 5, how do we solve for w?
  - If we don't know the area and we don't know the length but we want to solve for w (get w alone) what would we do?

Solving Formulas

- Perimeter of a Rectangle is P = 2l + 2w. Solve for w.

Solving Formulas

- Example: The formula for Area of a Sector is $A = \frac{1}{2} r^2 a$. Solve for a.

Solving Formulas

- Example: The formula for finding the resistance of electricity is $R = \frac{V}{I}$. Solve for I

Solving Formulas

- Example: Solve the Pythagorean Theorem $a^2 + b^2 = c^2$ for a.

Try Yourself

- 1) Solve the formula for Volume of a rectangular solid, V = lwh, for h.
- 2) Solve the formula for converting Celcius to Fahrenheit, $F = \frac{9C}{5} + 32$, for C.

Try Yourself

- 3) The formula to find the taper per inch of a piece of work is $T = \frac{D-d}{L}$. Solve for D.

Translating English to Algebra

- What are words that mean…
  - Addition
  - Subtraction
  - Multiplication
  - Division
  - Equals

Setting up Word Problems

- Try to directly copy the sentence, using operation signs and variables instead of the words.
- Read through the problem at least once first before you try to translate.
- Example: The diameter of a circle is twice the radius.
- Example: The radius of a circle is half of the diameter.

Setting up Word Problems

- Example: The pitch diameter D of a spur gear is equal to the number of teeth on the gear divided by the pitch.
- Example: The volume of a cylinder is equal to ¼ of its height times π times the square of its diameter.

Try Yourself

- 1. The sum of two weights is 300 lbs.
- 2. The volume of a solid bar is equal to the product of the cross-sectional area and the length of the bar.
- 3. The weight of a metal cylinder is approximately equal to 0.785 times the height of the cylinder times the density of the metal times the square of the diameter of the cylinder.

Setting up and Solving Word Problems

- You need to cut a 20-ft bar into three pieces so that the longest piece is three times as long as each of the other two equal lengths. Find the length of each piece.

Setting up and Solving Word Problems

- With a certain material, you typically lose 1/8 of the initial product. If you need to end up with a 9 in piece, what is the length of material that you should start with.

Try Yourself

- 1) The perimeter of a rectangular sheet of metal should end up being 40 ft. If the length is three times the width, find the dimensions of the sheet.

Try Yourself

- 2) A new company that sells metals is trying to get your business. They are advertising a delivery charge of only $10. The most common metal you order, is an angle iron that is $15.50/length with this new company. The company you usually use has a $50 delivery charge, but the angle iron only costs $14/length. At how many lengths would the two companies cost the same for an order?

Scientific Notation

- A number is written in scientific notation in the form: $P \times 10^k$ where P is a number 1 or more and less than 10 and k is an integer.
- Are these numbers in scientific notation?
  - 4.56×10^4
  - 0.6×10^3
  - 1400×10^{-3}
  - 1×10^{-5}
  - 10.68×10^2

Converting from Decimal Form to Scientific Notation

- Move the decimal place until you have one digit (beside 0) in front of the decimal point.
- If you moved the decimal to the left, k is positive.
- If you moved the decimal to the right, k is negative.
- Examples: Change to scientific notation
  - 20,960,000
  - 0.0000482

Converting from Scientific Notation to Decimal Form

- Move the decimal place k times and drop the x 10^k.
- Move the decimal point to the right if k is positive.
- Move the decimal point to the left if k is negative.
- Keep in mind, positive k means we are dealing with a very large number and negative k means we are dealing with a very small number.
- Examples: Change to decimal form
  - 7.3×10^{-4}
  - 2.18×10^8

Try Yourself

- 1) Change 4.56×10^{-3} to decimal form.
- 2) Change 948,800,000 to scientific notation.

Scientific Notation and Basic Operations

- We'll mostly use our calculators for doing calculations with scientific notation. The book goes into more detail.
- Example: 93,480,000 × 1,200,000
- Example: 0.041 ÷ 0.000053

Try Yourself

- 1) 0.00067÷86,000,000
- 2) 4,500 × 0.0000091